1. Given a number x_n with $x_n \geq -40$ set $x_{n+1} = \sqrt{x_n + 40}$.
   (a) (10 pts) Show by mathematical induction that if $x_1 = -39$, then $x_{n+1} > x_n$ for all integers $n \geq 1$.
   (b) (10 pts) Show by mathematical induction that if $x_1 = 24$, then $x_{n+1} < x_n$ for all integers $n \geq 1$.

2. (15 pts) Show by induction that
$$\frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdots \frac{2n+1}{2n+2} \leq \frac{1}{\sqrt{3n+4}}$$

3. (10 pts) Determine whether $(2+\sqrt{3})^{2/3}$ is a rational number, and explain your reasoning.

4. (10 pts) Find all rational solutions, if any, to the equation
$$9x^4 - 12x^3 + 13x^2 - 12x + 4 = 0$$
   Explain your reasoning.

5. (10 pts) Show by induction using the triangle inequality that
$$|a_1 + a_2 + \cdots + a_n| \leq |a_1| + |a_2| + \cdots + |a_n|$$
   for $a_i \in \mathbb{R}$ and all $n \geq 2$.

6. Show that for $a, b \in \mathbb{R}$ we have
   (a) (10 pts) $|a+b| + |a-b| = 2\max\{|a|, |b|\}$.
   (b) (10 pts) $\big||a+b| - |a-b|\big| \leq 2\min\{|a|, |b|\}$.

7. (15 pts) Given nonempty subsets A and B of $\mathbb{R}$ define the set $A - B$ by
$$A - B = \{a - b \colon a \in A, b \in B\}$$
   State and prove a formula for $\inf(A-B)$ in terms of $\inf(A), \sup(A), \inf(B)$, and $\sup(B)$.