MATH 3150 – HOMEWORK 1

Problem 1. Prove the following proposition.

Proposition. *In an ordered field, the following properties hold:*

(i) **Unique identities.** *If $a + x = a$ for every a, then $x = 0$. If $a \cdot x = a$ for every a, then $x = 1$.*
(ii) **Unique inverses.** *If $a + x = 0$, then $x = -a$. If $ax = 1$, then $x = a^{-1}$.*
(iii) **No divisors of zero**. *If $xy = 0$, then $x = 0$ or $y = 0$.*
(iv) **Cancellation for addition.** *If $a + x = b + x$ then $a = b$. If $a + x \leq b + x$, then $a \leq b$.*
(v) **Cancellation for multiplication.** *If $ax = bx$ and $x \neq 0$, then $a = b$. If $ax \geq bx$ and $x > 0$, then $a \geq b$.*
(vi) $0 \cdot x = 0$ *for every x.*
(vii) $-(-x) = x$ *for every x.*
(viii) $-x = (-1) \cdot x$ *for every x.*
(ix) *If $x \neq 0$, then $x^{-1} \neq 0$ and $(x^{-1})^{-1} = x$.*
(x) *If $x \neq 0$ and $y \neq 0$, then $xy \neq 0$ and $(xy)^{-1} = x^{-1}y^{-1}$.*
(xi) *If $x \leq y$ and $0 \leq z$, then $xz \leq yz$. If $x \leq y$ and $z \leq 0$, then $yz \leq xz$.*
(xii) *If $x \leq 0$ and $y \leq 0$, then $xy \geq 0$. If $x \leq 0$ and $0 \leq y$, then $xy \leq 0$.*
(xiii) $0 < 1$.
(xiv) *For any x, $x^2 \geq 0$.*

Problem 2. Give an example of a field with only three elements. Prove that it cannot be made into an ordered field.

Problem 3. Show that $3^n/n!$ converges to 0.

Problem 4. Let $x_n = \sqrt{n^2+1} - n$. Compute $\lim_{n\to\infty} x_n$.