Prof. Alexandru Suciu

MATH 3175 **Group Theory** **Spring 2024**

Homework 5

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1. Let G_1 and G_2 be two groups, with identities e_1 and e_2, respectively. Let $G = G_1 \times G_2$ and let $H = \{(g_1, g_2) \in G_1 \times G_2 : g_2 = e_2\}$. Show that

(i) H is a normal subgroup of G.

(ii) $H \cong G_1$.

(iii) $G/H \cong G_2$.

2. Let H be a subgroup of G, and define its *normalizer* as $N(H) := \{g \in G : gHg^{-1} = H\}$.

(i) Show that $N(H)$ is a subgroup of G.

(ii) Show that the subgroups of G that are conjugate to H are in one-to-one correspondence with the left cosets of $N(H)$ in G.

3. Given a group G and an element $a \in G$, we define the *centralizer of* a to be the set $C(a)$ of elements $x \in G$ that commute with a; that is, $C(a) := \{g \in G : ga = ag\}$.

(i) Show that $C(a)$ is a subgroup of G.

(ii) Show that $\langle a \rangle \subseteq C(a)$.

(iii) Show that $Z(G) \subseteq C(a)$.

4. Let $G = \mathbb{Z}_4 \times \mathbb{Z}_6$. Compute the factor groups $G/\langle(2, 3)\rangle$ and $G/\langle(3, 3)\rangle$. (In each case, write the result in terms of known finite groups, and explain your answer.)

5. Let G be a group of order 35. Suppose G has precisely one subgroup of order 5 and one subgroup of order 7. Show that G is a cyclic groups.