Prof. Alexandru Suciu

MATH 3175 | **Group Theory** | **Fall 2010**

Quiz 6

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1. Let H be set of all 2×2 matrices of the form $\begin{bmatrix} a & 0 \\ c & d \end{bmatrix}$, with $a, c, d \in \mathbb{Z}$ and $ad = \pm 1$.

(a) Show that H is a subgroup of $\mathrm{GL}_2(\mathbb{Z})$.

(b) Is H a normal subgroup of $\mathrm{GL}_2(\mathbb{Z})$?

2. Let $G = U(16)$, and $H = \{1, 15\}$.

(a) List the elements of G/H.

(b) Compute the Cayley table for this group.

3. Let $G = \mathbb{Z}_4 \oplus \mathbb{Z}_2$, and consider the subgroup $H = \{(0,0), (2,0), (0,1), (2,1)\}$.

(a) Identify the group H.

(b) Show that H is a normal subgroup of G.

(c) Identify the group G/H.

4. Let $\mathbb{R}^*$ be the multiplicative group of non-zero real numbers, and let $\phi\colon \mathbb{R}^* \to \mathbb{R}^*$ be the function given by $f(x) = x^2$.

(a) Show that ϕ is a homomorphism.

(b) Find $\ker(\phi)$ and $\operatorname{im}(\phi)$.

5. Suppose $\phi\colon \mathbb{Z}_{20} \to \mathbb{Z}_{12}$ is a homomorphism with $\phi(3) = 9$.

(a) Determine $\phi(x)$, for all $x \in \mathbb{Z}_{20}$.

(b) Find $\ker(\phi)$ and $\operatorname{im}(\phi)$.

6. Show that there is no surjective homomorphism from $\mathbb{Z}_{27} \oplus \mathbb{Z}_3$ onto $\mathbb{Z}_9 \oplus \mathbb{Z}_9$.