Prof. Alexandru Suciu

MATH 3175 **Group Theory** **Fall 2010**

Quiz 5

---

1. List all the elements of $\mathbb{Z}_2 \oplus \mathbb{Z}_8$, and compute their orders.

2. Show that the group $U(9)$ is isomorphic to the direct product $\mathbb{Z}_2 \oplus \mathbb{Z}_3$, by describing *explicitly* an isomorphism $\phi\colon U(9) \to \mathbb{Z}_2 \oplus \mathbb{Z}_3$.

3. Consider the group $G = S_3 \oplus \mathbb{Z}_6$.

(a) Determine the set of orders of elements in G, that is, the set $\{|g| \mid g \in G\}$.

(b) Prove that G is *not* cyclic.

4. How many elements of order 7 are there in $\mathbb{Z}_{70} \oplus \mathbb{Z}_{490}$?

5. List all abelian groups (up to isomorphism) of order 72. Write each such group as a direct product of cyclic groups of prime power order.

6. Let G be an abelian group of order 108. Suppose that G has exactly eight elements of order 3, and one element of order 2. Determine the isomorphism class of G.