Prof. Alexandru Suciu

MATH 3175 | **Group Theory** | **Fall 2010**

Quiz 1

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1. Consider the integers $a = 18$ and $b = 27$.

(i) Find $d = \gcd(18, 27)$ and $\ell = \operatorname{lcm}(18, 27)$.

(ii) What is the relationship between a, b, d, and ℓ predicted by the general theory? Verify this relationship holds in this situation.

(iii) Find a pair of integers s and t such that $18s + 27t = d$.

(iv) Find the general solution for all the pairs of integers s and t such that $18s + 27t = d$.

2. The following Latin square is the Cayley table of a group:

	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

(i) Verify associativity for the non-identity elements in the group.

(ii) Is the group abelian? Why, or why not?

(iii) What are the inverses of a, b, and c, respectively?

(iv) Is the inverse of ab equal to ba? Why, or why not?

3. Show that the following identities hold in any group. Explain your reasoning.

(i) $(a^{-1})^{-1} = a$.

(ii) $(a^{-1}ba)^3 = a^{-1}b^3a$.

4. Consider the group $U(12)$.

(i) List all the elements in $U(12)$, and write down the Cayley table for the group.

(ii) For each element a in $U(12)$, indicate what is a^{-1}.

5. Consider the following two matrices, viewed as elements in the group $\mathrm{GL}_2(\mathbb{Z}_7)$:

$$A = \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

(i) Find the inverse of the A in $\mathrm{GL}_2(\mathbb{Z}_7)$.

(ii) Compute the products $A \cdot B$ and $B \cdot A$. Are they the same, or not?

(iii) Is the group $\mathrm{GL}_2(\mathbb{Z}_7)$ commutative? Why, or why not?

6. Let G a group such that $(ab)^{-1} = a^{-1}b^{-1}$, for all a and b in G. Prove that G is abelian.