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MATH 3175 | **Group Theory** | **Fall 2010**

Practice Quiz 1

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1. Let $d = \gcd(20, 24)$.

(a) Find d.

(b) Find a pair of integers s and t such that $20s + 24t = d$.

(c) Find the general solution for all the pairs of integers s and t such that $20s+24t = d$.

2. Suppose $a = p_1^4 p_2^5 p_3$ and $b = p_2^3 p_3^9 p_4 p_5^8$, where $p_1, \dots, p_5$ are distinct primes.

(a) Find $\gcd(a, b)$.

(b) Find $\operatorname{lcm}(a, b)$.

(c) Check that $\gcd(a, b) \cdot \operatorname{lcm}(a, b) = ab$.

3. Determine whether the following Latin square is the Cayley table of a group. If that's not the case, give a reason why not. If that's the case, give an example of a group whose Cayley table is this one.

	e	a	b	c	d
e	e	a	b	c	d
a	a	b	d	e	c
b	b	d	c	a	e
c	c	e	a	d	b
d	d	c	e	b	a

4. Determine whether the following Latin square is the Cayley table of a group. If that's not the case, give a reason why not. If that's the case, give an example of a group whose Cayley table is this one.

	e	a	b	c	d
e	e	a	b	c	d
a	a	c	e	d	b
b	b	d	c	a	e
c	c	e	d	b	a
d	d	b	a	e	c

5. Consider the group $U(8)$.

(a) List all the elements in this group.

(b) Write down the Cayley table for this group.

(c) What is the (multiplicative) inverse of 7 in $U(8)$?

6. Consider the group $U(10)$.

(a) List all the elements in this group.

(b) Write down the Cayley table for this group.

(c) What is the (multiplicative) inverse of 7 in $U(10)$?

(d) Is the Cayley table for $U(10)$ the same as the one for $U(8)$, up to relabeling the elements?

7. Consider the matrix $A = \begin{pmatrix} 2 & 4 \\ 2 & 1 \end{pmatrix}$ in $\mathrm{GL}_2(\mathbb{Z}_{13})$. Find A^{-1}.

8. Consider the group $\mathrm{SL}_2(\mathbb{Z}_3)$.

(a) List all the elements in this group.

(b) Find two matrices in $\mathrm{SL}_2(\mathbb{Z}_3)$ which do not commute.

(c) Find two (distinct) matrices in $\mathrm{SL}_2(\mathbb{Z}_3)$ which do commute.

9. Let G a group with the following property: Whenever $a, b, c \in G$ and $ab = ca$, then $b = c$. Prove that G is abelian.

10. Let G a group, such that the square of any element is the identity. Prove that G is abelian.