For each problem be sure to explain the steps in your argument and fully justify your conclusions.

1. For the power series $\sum_{n=1}^{\infty} \frac{2^{2n}}{(3^{3n})\sqrt{n}} x^n$,
   (a) (9 pts) Find the radius of convergence.
   (b) (9 pts) Find the exact interval of convergence.

2. Let $f_n(x) = \frac{3n + 1 - \sin(x)}{2n + \cos(x)}$.
   (a) (9 pts) Show that (f_n) converges uniformly on $\mathbb{R}$. *Hint*: First decide what the limit function is and then show that convergence is uniform.
   (b) (9 pts) Using your result in part (a) and results in the text, determine $\lim_{n\to\infty} \int_a^b f_n(x)\,dx$ for $a < b$. Be sure to cite any results you use to justify your answer.

3. Let $f_n(x) = n^2 x e^{-nx^2}$.
   (a) (9 pts) Show that the sequence (f_n) converges pointwise on $\mathbb{R}$ and determine the function $f = \lim_{n\to\infty} f_n$.
   (b) (9 pts) Show that f_n does *not* converge uniformly on any interval containing 0.
   (c) (9 pts) Show that f_n does converge uniformly on any interval of the form $[a, \infty)$ with $a > 0$.

4. Let $f_n(x) = \sqrt{x} + \frac{1}{\sqrt{n}}$ and $f(x) = \sqrt{x}$, for $x \in [0, \infty)$.
   (a) (9 pts) Show that f_n converges to f uniformly on $[0, \infty)$.
   (b) (9 pts) Show that f_n^2 converges to f^2 pointwise on $[0, \infty)$.
   (c) (9 pts) Show that f_n^2 does *not* converge uniformly to f^2 on $[0, \infty)$.

5. (10 pts) Show that $\sum_{n=1}^{\infty} \frac{\sin(\sqrt{n}x)}{n^{3/2}}$ converges uniformly on $\mathbb{R}$ to a continuous function.