Be sure to include your reasoning in your answers to the following questions.

1. (a) (10 pts) Let (s_n) be a sequence such that
$$|s_{n+1} - s_n| < \frac{1}{n^{3/2}} \quad \text{for all } n \in \mathbb{N}$$
Prove that (s_n) is a Cauchy sequence and hence a convergent sequence.

   (b) (10 pts) Let (s_n) be a sequence such that
$$|s_{n+1} - s_n| < \frac{1}{n^{2/3}} \quad \text{for all } n \in \mathbb{N}$$
Show by means of an example that the sequence (s_n) may **not** converge.

2. Consider the sequence (x_n) with terms $x_n = (1 - 1/n)\cos(n\pi/4)$.
   (a) (10 pts) Write out the first 10 terms in this sequence
   (b) (10 pts) Give an example of a monotonic subsequence of (x_n).
   (c) (10 pts) Give the $\limsup x_n$ and $\liminf x_n$

3. (10 pts) Let (x_n) be a sequence with $\lim x_{2n} = 1$ and $\lim x_{2n+1} = 5$. Show that every convergent subsequence of x_n converges to either 1 or 5.

4. (10 pts) Let (x_n) and (y_n) be two bounded sequences of non-negative numbers. Show that
$$\liminf(x_n y_n) \geq \liminf(x_n) \cdot \liminf(y_n).$$

5. For each of the following series, determine whether the series converges or diverges. Justify your answers.
   (a) (10 pts) $\sum \frac{1}{n \ln(n)^3}$
   (b) (10 pts) $\sum_{n=2}^{\infty} \frac{n^2 + 2n + 7}{2^n - 1}$
   (c) (10 pts) $\sum (1 + 2/n)^n$