MATH 3150 Midterm Wednesday, October 25, 2023

Please **print** your name ________________________________

1. (14 pts) Use the definitions of $\limsup, \liminf$, and Cauchy sequence to show that if $\limsup x_n = \liminf x_n$, then (x_n) is a Cauchy sequence.

2. (13 pts) Let $f\colon \mathbb{R} \to \mathbb{R}$ be a continuous function, and assume that there is a number $a > 0$ with the property that $f(x) \geq a$ for all $x \in \mathbb{R}$. Use the $\delta - \epsilon$ definition of continuity to show that $1/f(x)$ is continuous for all $x \in \mathbb{R}$.

3. For each series below, determine whether the series converges or diverges. Be sure to name any tests that you use.

   (a) (10 pts) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{3^n}$

   (b) (10 pts) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$

4. Let (s_n) be the sequence with $s_1 = 11$ and $s_{n+1} = \dfrac{3}{4}(s_n + 1)$

   (a) (10 pts) Show that the sequence converges

   (b) (10 pts) Find the limit of the sequence

5. (a) (10 pts) Let $f\colon [1,4] \to \mathbb{R}$ be a continuous function such that $f(1) > 1$ and $f(4) < 2$. Show that there is a number $c \in [1,4]$ such that $f(c) = \sqrt{c}$.

   (b) (10 pts) Give an example of a (discontinuous) function $f\colon [1,4] \to \mathbb{R}$ such that $f(1) > 1$ and $f(4) < 2$ for which the equation $f(c) = \sqrt{c}$ has no solution $c \in [1,4]$.

6. (13 pts) Suppose $f\colon \mathbb{R} \to \mathbb{R}$ is a function which is *not* uniformly continuous on $\mathbb{R}$. Show that there is a particular $\epsilon_0 > 0$ and two sequences (x_n) and (y_n) such that

$$|x_n - y_n| \to 0 \quad \text{but} \quad |f(x_n) - f(y_n)| \geq \epsilon_0.$$