In your work on the following problems you may use the theorems about limits in section 9 of the text.

1. (17 pts) Find a function $f(\epsilon)$ defined for $\epsilon > 0$ with the property that
$$\left|\frac{3n+5}{7n-11} - \frac{3}{7}\right| < \epsilon \quad \text{for all } n \in \mathbb{N} \text{ with } n > f(\epsilon)$$

2. (17 pts) Find $\lim \sqrt{4n^2+3n+2} - 2n$

3. (17 pts) The squeeze theorem states that if $a_n \le x_n \le b_n$ for all $n \in \mathbb{N}$ and $\lim a_n = \lim b_n = L$, then the sequence (x_n) converges to L. Use the $N - \epsilon$ definition of limit to prove the squeeze theorem.

4. (17 pts) Use the $N - \epsilon$ definition of limit to show that the sequence with terms $x_n = \cos\left(\frac{n\pi}{3}\right)$ does not converge.

5. Let $x_1 = 2$ and $x_{n+1} = \frac{6x_n^2 + 1}{5}$ for $n \ge 1$.
   (a) (5 pts) Show that, if $a = \lim x_n$, then $a = \frac{1}{2}$ or $a = \frac{1}{3}$.
   (b) (5 pts) Does $\lim x_n$ exist?
   (c) (5 pts) Discuss the apparent contradiction between parts (a) and (b).

6. (17 pts) Let (x_n) be a convergent sequence. Suppose that $x_n \ge a$ for all but finitely many n. Show that $\lim x_n \ge a$.