

1. Given a number x_n with $x_n \geq -13$ set $x_{n+1} = \sqrt{x_n + 13}$.
 - (a) (10 pts) Show by mathematical induction that if $x_1 = -13$, then $x_{n+1} > x_n$ for all integers $n \geq 1$.
 - (b) (10 pts) Show by mathematical induction that if $x_1 = 12$, then $x_{n+1} < x_n$ for all integers $n \geq 1$.

2. (20 pts) Use the Rational Zeros Theorem to find all rational solutions, if any, to the equation
$$p(x) = 3x^4 + x^3 + 4x^2 + 2x - 4 = 0.$$
Explain your reasoning.

3. Determine whether the following numbers are rational or irrational. In each case, explain your reasoning.
 - (a) (10 pts) $\sqrt{6 + \sqrt{5}}$.
 - (b) (10 pts) $\sqrt{6 + 2\sqrt{5}} - \sqrt{5}$.

4. (20 pts) Let x be a real number. Show that if $|x - 1| < 1$, then $|x^2 - 1| < 3$.

5. (20 pts) Given nonempty subsets A and B of \mathbb{R} , with $A \cap B$ not equal to the empty set, prove directly from the definition of inf and sup that

$$\inf A \leq \inf(A \cap B) \leq \sup A$$