TOPOLOGY OF HYPERPLANE ARRANGEMENTS

LECTURES AT THE RESEARCH SCHOOL ON SINGULARITIES AND APPLICATIONS
UNIVERSITY OF LILLE, FRANCE

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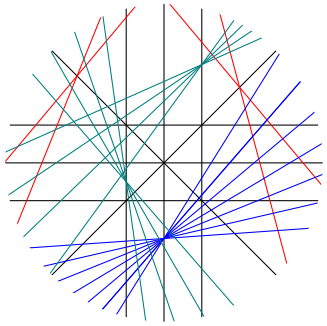
Abstract. Much of the fascination with arrangements of complex hyperplanes comes from the rich interplay between the combinatorics of the intersection lattice, the algebraic topology of the complement and its Milnor fibration. A key bridge between these objects is provided by the geometry of two sets of algebraic varieties associated to the complement: the resonance varieties of the cohomology ring and the characteristic varieties of the fundamental group. I will discuss some recent advances in our understanding of these topics, illustrating with concrete examples aided by computer computations.



Lecture 1. I will start by introducing the basic objects of study for these lectures: complex hyperplane arrangements, together with their intersection lattice, the cohomology ring and the fundamental group of the complement, and the Milnor fibration. Several classes of arrangements will be discussed; the examples will be illustrated with computer-aided computations, using the symbolic algebra packages Macaulay 2 and GAP.

Lecture 2. This lecture will be devoted to the cohomology jump loci of the complement of an arrangement: the resonance varieties, which only depend on the cohomology ring (and thus, are combinatorial in nature), and the characteristic varieties, which only depend on the fundamental group (in degree 1). A key role in the description of these varieties is played by the multinets on the sub-arrangements of the given arrangement. I will

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conclude with applications to the calculation of the homology of the Milnor fiber (once again illustrating with computational examples), and discuss some open problems.

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