Lower central series, free resolutions, and homotopy Lie algebras of arrangements

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ABSTRACT

The two main objects in the topological study of hyperplane arrangements are the cohomology ring and the fundamental group of the complement. The ring admits a readily computable, combinatorial description, due to Orlik and Solomon. The (rational) graded Lie algebra associated to the lower central series of the group is also combinatorially determined, though no effective procedure for computing its ranks is known, except in the case when the OS-algebra is Koszul.

In the first part of the talk, I will describe recent work with Hal Schenck, in which we determine the first four LCS ranks of the group from the Betti numbers of the free resolution of the OS-algebra over the exterior algebra. For several classes of arrangements, we make precise conjectures, expressing the LCS ranks in terms of known polynomials, and verify those conjectures in low ranks.

To a hyperplane arrangement $\mathcal{A}$, Dan Cohen, Fred Cohen and Miguel Xicoténcatl associate a sequence of "redundant" subspace arrangements, $\mathcal{A}^k$. For fiber-type arrangements, they show that the homotopy Lie algebra of $\mathcal{A}^k$ equals (up to rescaling) the graded Lie algebra associated to the fundamental group of $\mathcal{A}$.

In the second part of the talk, I will describe recent work with Stefan Papadima, in which we extend this "Rescaling Formula" for (rational) homotopy groups, to an arbitrary space with Koszul cohomology ring, and its corresponding sequence of homological rescalings. If the starting space is formal, we further upgrade this formula to the level of Malcev completions, and Milnor-Moore groups.